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NAMA
NUMERACY
FOR ADVANCED
MANUFACTURING

"Space and shape" learning units: Spatial visualisation

Theory and sample tasks

Sample version – for web view

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SPACE AND SHAPE - Spatial visualization

Often a worker must be able to imagine how an object will look like if she puts it in another way, or if she will look at it from another side. This ability is associated with making mental manipulations - you need to be able to move or rotate the solid mentally, or its elements thereof. Many times this ability involves such manipulations, which in mathematics are described by transformations: axial symmetry, rotation, parallel shift. Intuitively, the effects of such transformations describe how the figure was lying before and after the manipulation.

In parallel transformation

In mirror symmetry (line symmetry)

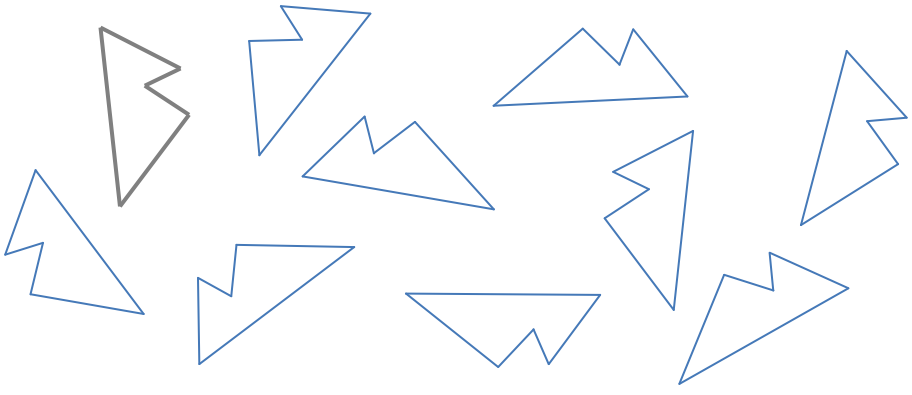
In rotation on any angle

In rotation on 180^0

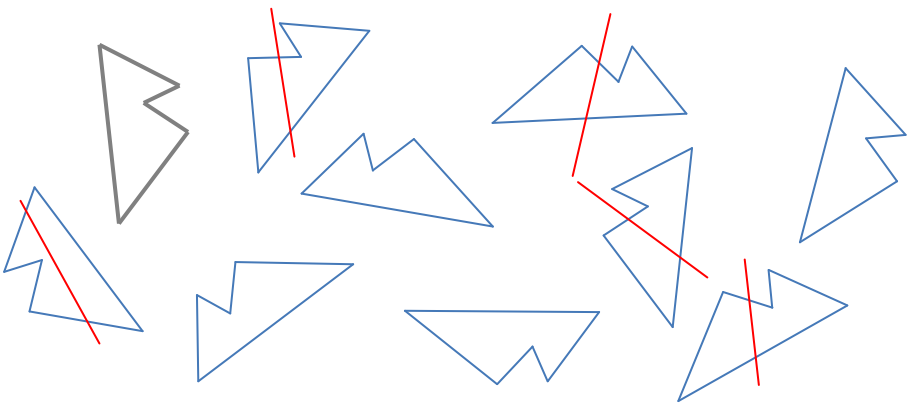
Exercises in acquiring such skills can start from manipulating in 2D. The introduction to 3D can be done by experimenting with a cube.

Manipulation in 2-D

Example:
This is a template of a figure (in black). Cross out all that figures, which are its symmetrical reflection (and after reflection their location on the plane might also change).



Answer:



Example:
Find the squares which are coloured in the same way:

1 2 3 4

Answer: 2 and 4

Example: Adjust the item to the card, from which it has been cut. Fill in the pairs in a table.

1 2 3 4 5

a b c d e

1	
2	
3	
4	
5	

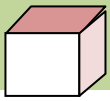
Answer:

1	b
2	d
3	e
4	a
5	c

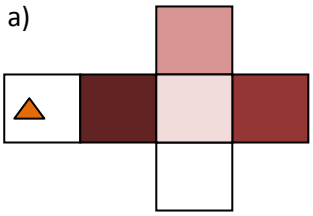
Manipulation in 3-D

Example:

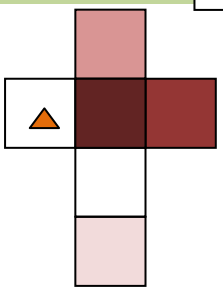
Which net was used for making this cube?



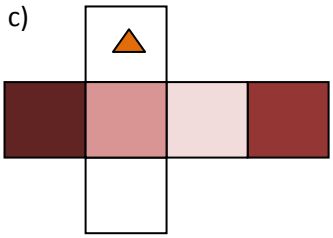
a)



b)

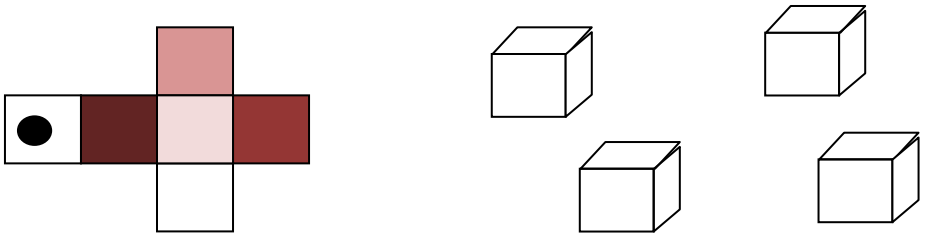


c)

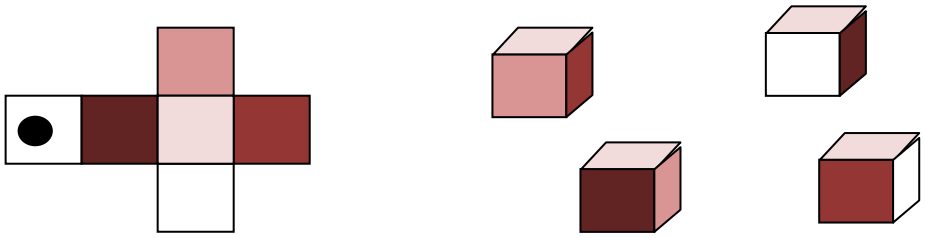


Answer:
c) Only this net has three walls coloured in such a way to be visible at a cube which, when assembled, will form a common vertex.

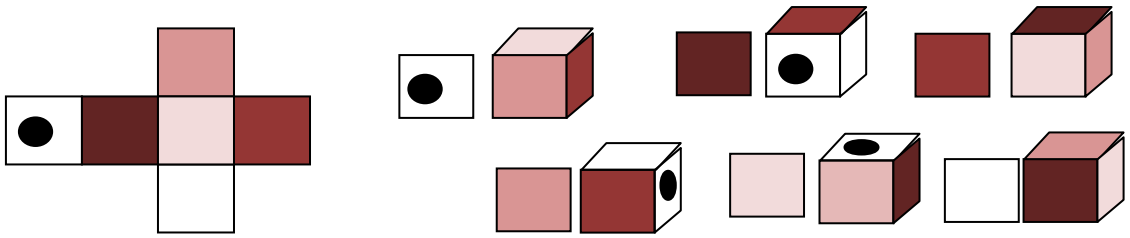
Example:
From the displayed net a cube has been sized and placed on the wall with a dot. Draw how it looks like from all four sides.



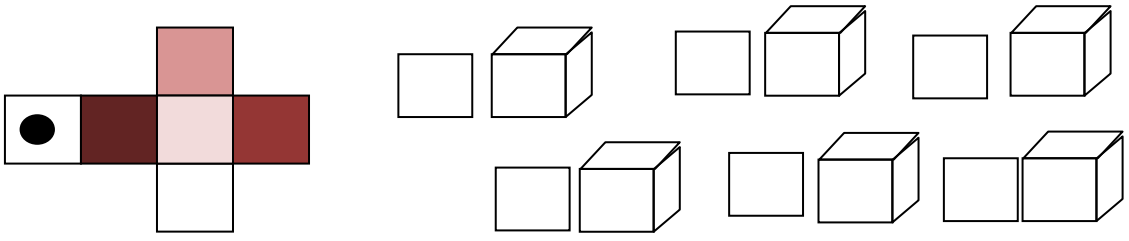
Answer:



Example:
How will the cube look like when placed on each of its six walls? Mark its base, then paint the view of the cube lying on the ground.
Is there only one solution to this task?

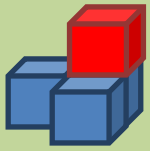


Answer: By putting the cube on the one side you can get four different views. A sample of the solution is shown below.



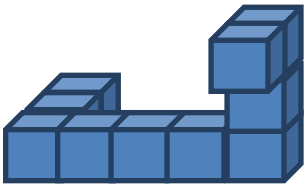
Example:

The structure shown consists of 4 cubes. Draw how it will look like, if a red cube will be placed in 4 different positions.

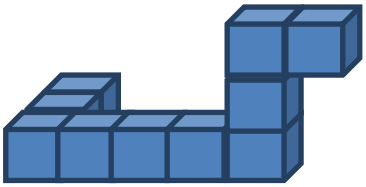


Answer: An indicative solution:

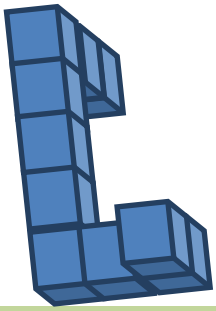
Example:
Which arrangement shows the model which is presented below?



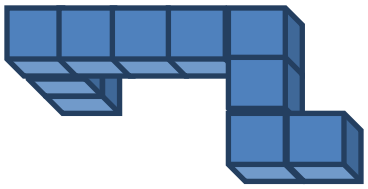
1)



2)



3)



Answer: 2).

SAMPLE TASKS

Task 1

Task 1 categorisation	
Mathematical field	Space and shape
Mathematical subfield	Units of measurement and scales Spatial visualisation
Formulae/data provided	1 µm = 0.001 mm
Numerical skills	Performing simple operations Estimating areas
Level of difficulty	1
Career /job role	Mechanical Technician, Mechanical Engineer, Mechatronics Technician, Mechatronics Engineer

If a machine cuts iron cast table tops from an iron surface of dimensions of 9200000 µm x 1250000 µm x 10000 µm, then how many table tops (of dimensions 1840 mm x 92 mm x 10 mm) will it cut?

Solution:

9200000 µm = 9200 mm = 9.20 m

1250000 µm = 1250 mm = 1.25 m

10000 µm = 10 mm

1840 mm =1.84 m

92 mm = 0.092 m

9.20 : 1.84 = 5

1.25 : 0.092 ≈ 13.59

5 x 13 = 65 table tops

The surface left is too small to cut more table tops.

Task 2

Task 2 categorisation	
Mathematical field	Space and shape
Mathematical subfield	Units of measurement and scales Spatial visualisation Application of numerical skills to solve geometrical problems
Formulae/data provided	1 µm = 0.001 mm
Numerical skills	Performing simple operations Estimating areas
Level of difficulty	2
Career /job role	Mechanical Technician, Mechanical Engineer, Mechatronics Technician, Mechatronics Engineer

a) If a machine cuts iron cast table tops from an iron surface of dimensions of 9200000 µm x 1250000 µm x 10000 µm, then how many table tops (of dimensions 1840 mm x 92 mm x 10 mm) will it cut?
b) Mr Smith ordered 180 iron cast table tops. How much will he pay if $1m^2$ of iron (of the 1cm gauge) costs 4 euros? The cost of production of one table top is 2 euros. The company is selling the table tops at 15% profit.

<u>Solution:</u>

a) 65 table tops (see task 1).

b) 180/65 = 2.76 ≈ 3 – Mr Smith has to pay for 3 iron cast surfaces

Total area of the surfaces: 11.5 m^2.

11.5 x 3 = 34.5 m^2 –area of 3 surfaces

34.5 x 4 = 138 euros – cost of material

2 x 180 = 360 euros –cost of production

360 + 138 = 498 euros

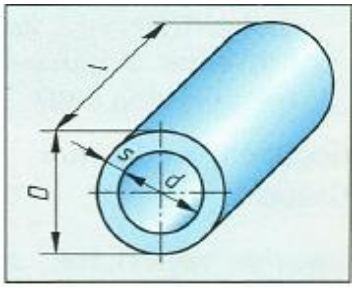
498 + 0.15 x 498 = 572.70 euros

Task 3

Task 3 categorisation	
Mathematical field	Space and shape
Mathematical subfield	Units of measurement and scales 2D-3D shapes and properties Spatial visualisation
Formulae/data provided	V_d – Volume of cylinder of diameter d: $V_d = \pi \cdot \left(\frac{d}{2}\right)^2 \cdot l$ Weight in relation to density ρ and volume V: $w = \rho \cdot V$
Numerical skills	Performing simple operations Calculating measures of weight and volume
Level of difficulty	1
Career /job role	CNC Operator

The permissible load capacity of a manual hoist is 500 kg. A CNC Operator has to calculate the weight of a pipe in order to check if by using the hoist he will be able to mount the pipe in the holder of the CNC machine.

Calculate the weight of a cast iron pipe given that its length is 3.5 m, its external diameter is 80 mm and the thickness of the wall is 15 mm (ρ = 7.2 g/cm^3).



Data:

D = 80 mm;

s = 15 mm;

l = 350 cm;

ρ = 7.2 g/cm^3

Solution:

V_D - Volume of cylinder of diameter D

V_d – Volume of cylinder of diameter d

$d = D - 2 \cdot s = 80 - 30 = 50 \text{ mm}$

$V_D = \pi \cdot 4^2 \cdot 350 = 5600\pi \text{ cm}^3$

$V_d = \pi \cdot 2.5^2 \cdot 350 = 2187.5\,\pi \text{ cm}^3$

V_p – Volume of pipe

$V_p = 5600\pi - 2187.5\pi = 3412.5\pi \approx 10715.25 \text{ cm}^3$

w –weight of pipe

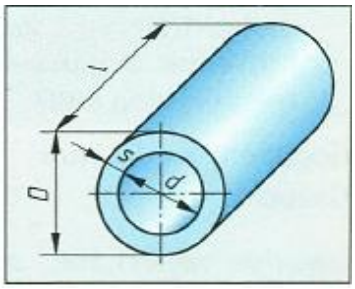
$w = \rho \cdot V$

$w = 7.2 \cdot 10715.25 = 77149.8 \text{ g} = 77.1498 \text{ kg}$

Task 4

Task 4 categorisation	
Mathematical field	Space and shape
Mathematical subfield	Units of measurement and scales 2D-3D shapes and properties Spatial visualisation
Formulae/data provided	V_d – Volume of cylinder of diameter d: $V_d = \pi \cdot \left(\frac{d}{2}\right)^2 \cdot l$ Weight in relation to density ρ and volume V: $w = \rho \cdot V$
Numerical skills	Performing simple operations Calculating measures of weight and volume
Level of difficulty	1
Career /job role	CNC Operator

What is the maximum length of a cast iron pipe in order to be lifted by manual hoist with a permissible load capacity of 500 kg? The external diameter of the pipe is 80 mm and the thickness of the wall is 15 mm (ρ = 7.2 g/cm^3).



Solution:

500 kg = 500000g

$500000 : 7.2 = 69444.(4) \approx 69444$ – maximum volume

$V_p = \pi \cdot 4^2 \cdot l - \pi \cdot 2.5^2 \cdot l = 9.75\,\pi\,l$

$9.75\,\pi\,l = 69444$

$\pi\,l \approx 7122.5$

$l_{max} \approx 2268.3$ cm = 22.683 m